Math Virtual Learning

Calculus AB

Differential Equations

April 16, 2020

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Lesson: April 16, 2020

Objective/Learning Target:

Students will use separation of variables to find a particular solution to a differential equation.

Warm-Up:

Watch Video: Review Separation of Variables

Read Article: Review Separation of Variables

Examples:

To find a particular solution using separation of variables we will just add an extra step to what we did for finding a general solution using separation of variables. Please refer back to yesterday's lesson (April 15) for review on that process.

Watch Video: Worked out Example of Separation of Variables to find Particular Solution

Examples:

Given the initial condition $y(0) = 1$, find the particular solution of the equation

$$xy\,dx + e^{-x^2}(y^2 - 1)\,dy = 0.$$

Solution Note that $y = 0$ is a solution of the differential equation—but this solution does not satisfy the initial condition. So, you can assume that $y \neq 0$. To separate variables, you must rid the first term of y and the second term of e^{-x^2}. So, you should multiply by e^{x^2}/y and obtain the following.

$$xy\,dx + e^{-x^2}(y^2 - 1)\,dy = 0$$

$$e^{-x^2}(y^2 - 1)\,dy = -xy\,dx$$

$$\int \left(y - \frac{1}{y}\right) dy = \int -xe^{x^2}\,dx$$

$$\frac{y^2}{2} - \ln|y| = -\frac{1}{2}e^{x^2} + C$$

From the initial condition $y(0) = 1$, you have $\frac{1}{2} - 0 = -\frac{1}{2} + C$, which implies that $C = 1$. So, the particular solution has the implicit form

$$\frac{y^2}{2} - \ln|y| = -\frac{1}{2}e^{x^2} + 1$$

$$y^2 - \ln y^2 + e^{x^2} = 2.$$

You can check this by differentiating and rewriting to get the original equation.

Examples:

Find the equation of the curve that passes through the point $(1, 3)$ and has a slope of y/x^2 at any point (x, y).

Solution Because the slope of the curve is given by y/x^2, you have

$$\frac{dy}{dx} = \frac{y}{x^2}$$

with the initial condition $y(1) = 3$. Separating variables and integrating produces

$$\int \frac{dy}{y} = \int \frac{dx}{x^2}, \quad y \neq 0$$

$$\ln|y| = -\frac{1}{x} + C_1$$

$$y = e^{-(1/x)+C_1} = Ce^{-1/x}.$$

Because $y = 3$ when $x = 1$, it follows that $3 = Ce^{-1}$ and $C = 3e$. So, the equation of the specified curve is

$$y = (3e)e^{-1/x} = 3e^{(x-1)/x}, \quad x > 0.$$

Practice:

1) Find the general solution of $\frac{dy}{dx} = 3x^2e^{-y}$ and the particular solution that satisfies the condition $y(0) = 1$

2) Solve the equation $\frac{dy}{dx} = \frac{y+1}{x-1}$ given the boundary condition: $y = 1$ at $x = 0$

Answer Key:

Once you have completed the problems, check your answers here.

1) General solution is $y = \ln(x^3 + A)$, and particular solution is $y = \ln(x^3 + e)$,

2) General solution is $y + 1 = k(x - 1)$, and particular solution is $y = -2x + 1$,

Additional Practice:

In your Calculus book read section 6.3 and complete problems 13, 15, 17, and 19 on page 429

Extra Practice with Answers